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A brane teaser

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Abstract: In this note we study the puzzle posed by two M5-branes intersecting on a string (or equivalently, a single M5-brane wrapping a holomorphic four-cycle in $\mathcal{C}^4$). It has been known for a while that this system is different from all other configurations built using self-intersecting M-branes; in particular the corresponding supergravity solution exhibits various curious features which have remained unexplained. We propose that the resolution to these puzzles lies in the existence of a non-zero two-form on the M5-brane world-volume.

Keywords: M-Theory, p-branes.

Contents

1. The puzzle in a nutshell

First things first. The puzzle is that an M5-brane wrapping a four-cycle holomorphically embedded in $\mathbb{C}^4$ is different from all other M-branes wrapped on holomorphic cycles [1]. This difference is manifest in several ways[1] and proves to be a special nuisance when we attempt to find the corresponding supergravity solution. In this note we will examine this unusual wrapped M5-brane, using the information encoded in its supergravity solution. However, before we attempt to resolve the puzzle of *why* this brane is so different, we will first describe briefly the ways in which it stands out from the crowd.

Often, a system of self-intersecting Mp-branes corresponds to the singular limit of a single Mp-brane wrapping a smooth cycle. More precisely speaking, this is the case when the cycle in question is described by factorisable embedding functions; each factor

[1] It was pointed out in [2] that an M5-brane wrapping a four-cycle is inconsistent because of anomalies

then specifies the world-volume of one constituent brane. For an Mp-brane wrapped on a holomorphic cycle, the configuration obtained in the limit when the cycle becomes singular is given by a number of planar Mp-branes intersecting orthogonally along $(p-2)$ spatial directions, as expected by the $(p-2)$ self intersection rule [3, 4].

Rules however, do have exceptions and in the M5-brane configuration under study here, the $(p-2)$-rule has finally met its Waterloo. There is no way in which an M5-brane wrapped on a holomorphic four-cycle in $\mathbb{C}^4$ can be realised as a system of orthogonally intersecting M5-branes, with each pair of branes intersecting in 3 (spatial) directions.

At least one pair of fivebranes with a string intersection must be included in order for the resulting configuration to 'smoothen out' into a single fivebrane wrapped on a holomorphic four-cycle in $\mathbb{C}^4$. The simplest intersecting brane realisation of this wrapped M5-brane is in fact given by a single pair of orthogonal fivebranes which share only one spatial direction, as shown below.

	0	1	2	3	4	5	6	7	8	9	10
M5	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$					
M5	$\times$	$\times$					$\times$	$\times$	$\times$	$\times$	

(1.1)

But the anarchy does not end here. This brane configuration, not yet happy with the havoc it has caused, proceeds to also turn the harmonic function rule on its head! The harmonic function rule [5], a prescription for constructing the supergravity solution for systems of intersecting branes, gets its name from the fact that the contribution of each constituent brane to the resulting solution can be expressed in terms of a harmonic function. One of the hallmarks of this rule was that the harmonic functions depend only on coordinates, which are simultaneously transverse to every brane in the system.

Based on experience, we would thus expect the supergravity solution for the configuration (1.1) to depend only on X^{10}. We find however, that the solution is completely indepent of this overall transverse direction and the harmonic functions depend instead on the relative transverse directions $(X^2 \dots X^9)$. This unparalleled behaviour is in itself enough to indicate that there is something special going on with this system.

All these factors put together make for a rather complicated situation; one which is not at all well understood. It is in an attempt to de-mystify this intersecting brane system that we turn to its 'smooth' version; an M5-brane wrapped on a holomorphic four-cycle in $\mathbb{C}^4$. We explore the corresponding supergravity solution, hoping to pick up clues which will help us uncover the underlying reason why this configuration breaks all the rules.

2. A tool kit

Before we can expect to answer a question, we must first understand exactly what it is that we are asking. In formulating our question more precisely, we will also get some valuable hints on where to begin looking for an answer. In this section, we will essentially collect tools which will help us in the process of both asking the question, and later, answering it. We first skim over some basic background which is relevant for the problem at hand.

We start with a short review of planar BPS M-branes and the corresponding supergravity solutions [6]. This allows us to set notation and also to remind the reader of some facts which will be used later. We then move on to the harmonic function rule, which enables us to build supergravity solutions for more complicated BPS brane configurations in M-Theory. Applying this rule to the system (1.1) we are lead to several contradictions. By exploring these contradictions we gain a deeper understanding of the issues we need to address.

2.1 Understanding the question

Half BPS planar M2-branes and M5-branes give rise to a large number of supersymmetric states in M-Theory. These states can be generated either by wrapping flat M-branes on supersymmetric cycles, or by building configurations of intersecting branes.

Supergravity solutions for planar M-branes: before we present the actual solutions for flat M-branes, we pause for a moment to discuss the general features we expect to these solutions to contain. Since the world-volume of a planar M-brane respects Poincare invariance, the metric describing the surrounding space-time should be independent of coordinates tangent to the M-brane. Also, the configuration is invariant under rotations in the transverse directions; this is reflected in the fact that the metric is a function of only the radial coordinate r in the transverse space. Under these conditions, the equation of motion for the field strength $d * F = 0$ reduces to the requirement that the function characterising the metric is in fact a *harmonic* function of r. All these features are manifest in the solutions given below.

Planar M5 and M2-branes are massive objects which are charged under the supergravity three-form. When placed in flat spacetime they deform it in such a way that the resulting background can be described as follows:

M5-brane	$ds^2 = H^{-1/3}\eta_{\mu\nu}dX^\mu dX^\nu + H^{2/3}\delta_{\alpha\beta}dX^\alpha dX^\beta$ and $F_{\alpha\beta\gamma\delta} = \frac{1}{2}\epsilon_{\alpha\beta\gamma\delta\rho}\partial_\rho H$ where $H = 1 + \frac{a}{r^3}$	(2.1)
M2-brane	$ds^2 = H^{-2/3}\eta_{\mu\nu}dX^\mu dX^\nu + H^{1/3}\delta_{\alpha\beta}dX^\alpha dX^\beta$ and $F_{\mu_0\mu_1\mu_2\alpha} = \frac{\partial_\alpha H}{2H^2}$ where $H = 1 + \frac{a}{r^6}$	

In the expressions above, X^μ denotes coordinates tangent to a brane and r is the radial coordinate in the transverse space spanned by coordinates X^α.

Planar M-branes, being half BPS objects, preserve 16 real spacetime supersymmetries. These correspond to the components of a spinor χ which satisfies the condition $\hat{\Gamma}_{\mu_0\mu_1\mu_2\mu_3\mu_4\mu_5}\chi = \chi$ for an M5-brane with worldvolume $X^{\mu_0}\dots X^{\mu_5}$, and $\hat{\Gamma}_{\mu_0\mu_1\mu_2}\chi = \chi$ for a flat M2-brane spanning directions $X^{\mu_0}X^{\mu_1}X^{\mu_2}$.

Harmonic function rule. A system of M-branes gives rise to a bosonic background which combines the individual effects of each constituent brane. If the branes are aligned in a certain way, the resulting background can still be a solution to the equations of motion of D=11 supergravity. Since the solution for a single brane is expressible in terms of a harmonic function, an extremal (no binding energy) BPS configuration of N branes is expected to have a supergravity solution characterised by N independent harmonic functions. The harmonic function rule [5] gives us a way in which to obtain the supergravity solution of an intersecting brane system by superposing the individual bosonic fields arising from each component brane.

Before we specify the rule, here is some notation: In the presence of an intersecting M-brane configuration, spacetime can naturally be 'divided' into three subspaces. The directions common to all constituent M-branes live in the common tangent subspace. The relative transverse subspace is spanned by directions which are tangent to at least one but not all of the planar M-branes (From the wrapped brane point of view, this is known as the embedding space, as it is where the supersymmetric cycle wrapped by the M-brane is embedded). Finally, there is the overall transverse subspace which spans directions transverse to all constituent branes in the intersecting brane system.
The field strength: the field strength components due to each constituent M-brane carry different indices, hence the field strength of the resulting intersecting brane configuration can be obtained merely by adding the individual field strengths corresponding to each M-brane.
The metric: assigning a harmonic function to each constituent M-brane, the metric for an orthogonally intersecting M-brane system can be written in the form

$$ds^2 = \sum_I f_I(X^\alpha)\ \eta_{IJ} dX^I dX^J \tag{2.2}$$

where I, J run over all spacetime and α is used to label the overall transverse directions. In analogy to the single brane case, the coefficient f_I contains a factor $H^{-1/3}$ due to each M5-brane (and $H^{-2/3}$ due to each M2-brane) whose world-volume includes the coordinate X_I. Similarly, every M5-brane lying transverse to X_I contributes a factor of $H^{2/3}$ (whereas every transverse M2 brane contributes $H^{1/3}$) to f_I.

An example: in order to illustrate the application of the harmonic function rule, consider the following system of two M5-branes intersecting in three spatial directions.

	0	1	2	3	4	5	6	7	8	9	10
M5	×	×	×	×	×	×					
M5	×	×	×	×			×	×			

The harmonic function rule then dictates the following metric

$$\begin{aligned} ds^2 \;=\; & H_1^{-1/3} H_2^{-1/3}(-dX_0^2 + dX_1^2 + dX_2^2 + dX_3^2) + H_1^{-1/3} H_2^{2/3}(dX_4^2 + dX_5^2) + \\ & + H_1^{2/3} H_2^{-1/3}(dX_6^2 + dX_7^2) + H_1^{2/3} H_2^{2/3}(dX_8^2 + dX_9^2 + dX_{10}^2) \end{aligned}$$

and field strength components

$$F_{67\alpha\beta} = \epsilon_{\alpha\beta\gamma} H_1 \qquad F_{45\alpha\beta} = \epsilon_{\alpha\beta\gamma} H_2$$

where the functions H_1 and H_2 depend only on X^α for $\alpha = 8, 9, 10$.

2.2 Spelling out the problem

Applying the harmonic function rule to the system (1.1) leads to the metric:

$$\begin{aligned} ds^2 =& H_1^{-1/3}H_2^{-1/3}(-dX_0^2+dX_1^2)+H_1^{-1/3}H_2^{2/3}(dX_2^2+dX_3^2+dX_4^2+dX_5^2)+ \\ &+H_2^{1/3}H_2^{-1/3}(dX_6^2+dX_7^2+dX_8^2+dX_9^2)+H_1^{2/3}H_2^{2/3}dX_{10}^2 \end{aligned} \tag{2.3}$$

The scarcity of overall transverse directions leads to several glaring problems. The most immediate issue is of course that if H_1 and H_2 are functions of X^{10} alone, then they cannot possibly be harmonic functions and still have the required fall-off at infinity. Moreover, it is completely unclear what the expressions for the field strength components should be; we obviously cannot proceed in analogy to the example worked out above.

A rather unusual solution was proposed in [7]. The form of the metric (2.3) was left unmodified, but the roles of the overall transverse and relative transverse directions were interchanged. Rather than depending on the overall tranverse direction, it was suggested that the functions in the metric anstaz depend instead on the relative transverse directions. In particular, H_1 was found to be a harmonic function of $X^6 \ldots X^9$, and H_2 was required to solve the flat space laplacian in $X^2 \ldots X^5$. Imposing this functional dependence did make it possible to compute a supergravity solution, but it did not shed any light on why, in this particular case, the harmonic functions behave in such a strange manner. This is the question which served as the motivation for the work presented in this note, and hence it is a question we will return to later.

2.3 Looking for an answer

Having now made the puzzle explicit, we begin our search for the resolution. We start by adding to our tool-kit of concepts a few basic ideas which will save us a lot of labour in our quest for an answer, and also provide us with the necessary vocabulary in which to formulate the results of our search.

Spinors on complex manifolds. Spinors on complex manifolds can be expressed as Fock space states, using the fact that the Clifford algebra in flat compex space resembles the algebra of fermionic creation and annihilation operators. More explicitly, the Clifford algebra in $\mathbb{C}^n$ takes the form

$$\{\Gamma_{z_i}, \Gamma_{\overline{z}_j}\} = 2\eta_{i\bar{j}} \,. \tag{2.4}$$

where $i, j = 1 \ldots n$ and we have defined the Γ matrices for a complex coordinate $z_j = x_j + iy_j$ as follows:

$$\begin{aligned} \Gamma_{z_j} &= \frac{1}{2}(\Gamma_{x_j} + i\Gamma_{y_j}) \\ \Gamma_{\overline{z}_j} &= \frac{1}{2}(\Gamma_{x_j} - i\Gamma_{y_j}) \end{aligned} \tag{2.5}$$

Declaring Γ_{z_i} to be creation operators and $\Gamma_{\overline{z}_j}$ to be annihilation operators, a Fock space can be generated by acting the creation operators on a vacuum. Because there are n creation operators, each state in the Fock space is labelled by n integers taking values 0 or 1 which correspond to its fermionic occupation numbers. It will later prove very useful to express Killing Spinors in this way.

The Fayyazuddin-Smith ansatz. By its very construction, the harmonic function rule does not extend to supergravity solutions for branes localised in the relative transverse directions. In an attempt to find supergravity solutions for such localised brane intersections Fayyazuddin and Smith [8] proposed a metric ansatz by applying symmetry based arguments to the background of a curved M-brane.

When an Mp-brane wraps a supersymmetric m-cycle, part of the world-volume of the brane will in general remain unwrapped. Poincare invariance is expected to hold along these $(p+1-m)$ unwrapped (common tangent) directions X^μ, implying that the metric should be independent of these coordinates. Rotational invariance in the overall transverse directions X_α leads to a diagonal metric in this subspace and further dictates that the undetermined functions in the metric ansatz depend only on $\rho = \sqrt{X_\alpha X^\alpha}$. A complex structure can be defined in the remaining (relative transverse) directions. The hermitean metric $G_{M\bar{N}}$ in this subspace cannot be constrained using the isometries of the brane configuration.

A metric incorporating all the above features takes the form

$$ds^2 = H_1^2 \eta_{\mu\nu} dX^\mu dX^\nu + 2G_{M\bar{N}} dZ^M dZ^{\bar{N}} + H_2^2 \delta_{\alpha\beta} dX^\alpha dX^\beta \tag{2.6}$$

and has come to be known as the Fayyazuddin-Smith metric ansatz.

Looking for supergravity solutions. The fact that a configuration preserves supersymmetry implies that the supersymmetric variation $\delta_\chi \Phi$ vanishes for all fields Φ when the variation parameter χ is a Killing spinor of the background.

Denoting flat indices in 11-dimensional spacetime by i, j and curved indices by I, J the bosonic part of the action for 11d supergravity can be written as

$$S = \int d^{11}X \sqrt{-G} \left\{ R - \frac{1}{12} F^2 - \frac{1}{432} \epsilon^{I_1 \dots I_{11}} F_{I_1 \dots I_4} F_{I_5 \dots I_8} A_{I_9 \dots I_{11}} \right\} \tag{2.7}$$

Throughout this note we will be dealing solely with bosonic backgrounds, fermionic fields have been set to zero and the supersymmetric variations of the metric and four-form field strength vanish identically. The only non-trivial requirements for supersymmetry preservation arise from the gravitino variation equation

$$\delta \Psi_I = \left(\partial_I + \frac{1}{4} \omega_I^{ij} \hat{\Gamma}_{ij} + \frac{1}{144} \Gamma_I{}^{JKLM} F_{JKLM} - \frac{1}{18} \Gamma^{JKL} F_{IJKL} \right) \chi \,. \tag{2.8}$$

We could also turn the logic around. By requiring $\delta_\chi \Psi = 0$ to hold for a given metric, we find a set of relations between the metric and field strength of the supergravity three-form which must be true if supersymmetry is to be preserved by the background. If the four-form obeying these relations is such that $dF = 0$ and $d \star F + \frac{1}{2} F \wedge F = 0$, then Einstein's equations are guaranteed to be satisfied and we have determined the bosonic components of a BPS solution to 11-dimensional supergravity.

This is the procedure followed in the proceeding section, to analyse an M5-brane wrapping a holomorphic 4-cycle in $\mathbb{C}^4$. We enforce supersymmetry preservation for a metric of the form proposed by Fayyazuddin and Smith. Expressing χ as a sum of Fock space states, $\delta\Psi_I = 0$ reduces to a sum of linearly independent relations each of which must be put to zero seperately. Satisfying these relations is the first step to finding the supergravity solution corresponding to the M-brane in question.

3. The mischief maker

Having a complete tool kit at our disposal, we are now in a position to break down our problem and study its various components. We begin by writing down the Fayyazuddin-Smith ansatz for the metric describing the supergravity background created by an M5-brane wrapping a holomorphic 4-cycle in $\mathbb{C}^4$. As can be seen from the discussion in the previous section, the relevant metric ansatz is

$$ds^2 = H_1^2 \eta_{\mu\nu} dX^\mu dX^\nu + 2G_{M\bar{N}} dZ^M dZ^{\bar{N}} + H_2^2 dy^2 \tag{3.1}$$

where Z^M denote the holomorphic coordinates s, u, v, w in $\mathbb{C}^4$ and y is the overall transverse direction. In order for Lorentz invariance to be preserved along the unwrapped directions of the brane's worldvolume, the functions H_1, H_2 and $G_{M\bar{N}}$ must be independent of $X_\mu = X^0$ and X^1.

3.1 Killing spinors

The amount of supersymmetry preserved by a p-brane with worldvolume $X^{M_0} X^{M_1} \dots X^{M_p}$ is given by the number of spinors which satisfy the equation [9]

$$\chi = \frac{1}{(p+1)!} \epsilon^{\alpha_0 \alpha_1 \dots \alpha_p} \Gamma_{M_0 M_1 \dots M_p} \partial_{\alpha_0} X^{M_0} \partial_{\alpha_1} X^{M_1} \cdots \partial_{\alpha_p} X^{M_p} \chi \tag{3.2}$$

where $\Gamma_{M_0 M_1 \dots M_p}$ is the completely anti-symmetrized product of $p+1$ eleven dimensional Γ matrices.

Consider now the metric (3.1). An 11-dimensional spinor in this background can be decomposed into spinors within and transverse to $\mathbb{C}^4$. The spinors on $\mathbb{C}^4$ can then be realised as linear combinations of Fock space states, as explained earlier. Using this construction, Killing spinors χ of the spacetime (3.1) can be expressed as a sum of terms of the form $\alpha \otimes \psi \equiv \alpha \otimes |n_s n_u, n_v, n_w\rangle$ where α is a three-dimensional spinor and n_z (for $z = s, u, v, w$) are the fermionic occupation numbers of the state, corresponding to the action of Γ_z on the Fock vacuum.

From (3.2) we see that the Killing spinors for an M5-brane wrapping a supersymmetric four-cycle in $\mathbb{C}^4$ are such that [10]

$$\epsilon^{abcd} \Gamma_{01} \Gamma_{m\bar{n}p\bar{q}} \partial_a X^m \partial_b X^{\bar{n}} \partial_c X^p \partial_d X^{\bar{q}} \chi = \chi \tag{3.3}$$

where the Γ_m are flat space Γ-matrices and $\sigma^a \dots \sigma^d$ are coordinates on the four-cycle. This can be simplified and expressed as follows

$$\Gamma_{01} \Gamma_{m\bar{n}p\bar{q}} \chi = (\eta_{m\bar{n}} \eta_{p\bar{q}} - \eta_{m\bar{q}} \eta_{p\bar{n}}) \chi \tag{3.4}$$

where $\eta_{m\bar{n}}$ is the flat space metric in $\mathbb{C}^4$. A solution to this equation is given by the Majorana spinor χ such that

$$\chi = \alpha \otimes (|0000\rangle + |1111\rangle) \tag{3.5}$$

where the chirality of the spinor α in the space-time transverse to $\mathbb{C}^4$ is fixed by the requirement that

$$\Gamma_{01}\ \alpha = \alpha \tag{3.6}$$

In order to count the number of supercharges preserved by this configuration, note that as a generic spinor χ would have had 32 complex components: 2 complex components come from the Dirac spinor α and there are 16 linearly independent Fock space states in $\mathbb{C}^4$. After the constraints (3.4) are imposed, only 2 of these 16 components survive and χ is left with 4 complex degrees of freedom. Determining the chirality of α cuts the degrees of freedom down by half and enforcing the Majorana condition cuts them down by a further half. The wrapped M5-brane thus preserves 1/16 of the spacetime supercharges, corresponding to the 2 spinors which satisfy the above conditions.

3.2 Supersymmetry preservation conditions

Contrary to the way things are normally done, we will consider for the moment that all possible components of the four-form field strength could in principle be turned on. Conventional wisdom dictates that since fivebranes couple to the supergravity three-form purely magnetically, only magnetic components of the field strength (i.e., those with indices purely transverse to the worldvolume) will be present. However, conventional wisdom hasn't exactly served us very well when it comes to this particular configuration, so for now we keep the electric components in the picture. The only arguments we admit at this point are those of symmetry. Components like $F_{\mu ABC}$ (where A, B, C take values in C^4 and y) would destroy the SO$(1,1)$ isometry expected of the solution and are thus set to zero.

We proceed to look for BPS solutions to $d = 11$ supergravity by demanding that the supersymmetry variation of the gravitino vanishes for the metric ansatz (3.1) and Killing spinor (3.5). This gives us a set of equations which can be solved to obtain expressions for the field strength components. In addition, we also find a constraint on the metric and a relation between the metric and the Killing spinor. These are given below.

Metric constraint. The metric is subject to the constraint

$$\partial\left[G^{-\frac{2}{3}} {H_1}^{-\frac{10}{3}} H_2 \omega \wedge \omega \wedge \omega\right] = 0\,. \tag{3.7}$$

Here, $\sqrt{G}$ denotes the determinant of the hermitean metric in the complex subspace, and $\omega = iG_{M\bar{N}} dZ^M dZ^{\bar{N}}$ is the associated two-form.

Killing spinors. The Killing spinor χ is specified through (3.5) once we determine α. We already know that α is proprotional to a constant spinor whose chirality is fixed through (3.6). All that needs to be found in order to uniquely specify α is the factor multiplying the constant spinor. Supersymmetry preservation dictates this factor be such that

$$24\partial_{\bar{R}} \ln \alpha = \partial_{\bar{R}} \ln G + 20\partial_{\bar{R}} \ln H_1 \,. \tag{3.8}$$

Field strength components. The gravitino variation equations set the components F_{01MN} and $F_{M\bar{N}\bar{P}\bar{Q}}$ to zero identically. The remaining field strength components can roughly be catagorized as follows. To begin with, there are components with indices lying along the world-volume of the brane; these will be referred to in the following as electric type components, since they imply that the wrapped M5-brane also couples *electrically* to the supergravity three-form, defying intuition. These components are:

$$F_{01\bar{R}y} = \frac{1}{2}\partial_{\bar{R}}({H_1}^2 H_2) \tag{3.9}$$

and

$$F_{01M\bar{N}} = \frac{H_1}{2H_2}\partial_y(H_1 G_{M\bar{N}}) \,. \tag{3.10}$$

Then there are components with a more familiar structure. These have indices in the (relative and overall) transverse directions only, indicating that the fivebrane couples magnetically to the supergravity three-form, as expected. By solving the gravitino variation equations, we are able to determine some of these components completely and to impose contraints on the others. Two components for which we are able to obtain explicit expressions are

$$F_{suvw} = -\frac{\sqrt{G}}{4H_2}[\partial_y \ln G + 8\partial_y \ln H_1] \tag{3.11}$$

and

$$F_{MP\bar{Q}\bar{N}} = \frac{1}{6H_2 H_1^5}\partial_y \left[H_1^5 G_{P\bar{Q}} G_{M\bar{N}} - G_{M\bar{Q}} G_{P\bar{N}})\right] . \tag{3.12}$$

The component $F_{M\bar{N}\bar{P}y}$ is determined in terms of F_{MNPy} by the following equation

$$\begin{aligned} G_{M\bar{Q}} &[9\partial_{\bar{P}} \ln H_2 - \partial_{\bar{P}} \ln G - 8\partial_{\bar{P}} \ln H_1] - G_{M\bar{P}} \left[9\partial_{\bar{Q}} \ln H_2 - \partial_{\bar{Q}} \ln G - 8\partial_{\bar{Q}} \ln H_1\right] = \\ &= 9\left[\partial_{\bar{Q}} G_{M\bar{P}} - \partial_{\bar{P}} G_{M\bar{Q}}\right] - 6{H_2}^{-1}\epsilon^{RN}{}_{\bar{P}\bar{Q}} F_{MRNy} - 18{H_2}^{-1} F_{M\bar{P}\bar{Q}y} \,. \end{aligned} \tag{3.13}$$

The only hitch is that F_{MNPy} cannot be fixed yet. However, we know that it is subject to

$${H_2}^{-1}\epsilon^{MNP}{}_{\bar{R}} F_{MNPy} - \partial_{\bar{R}} \ln G - 8\partial_{\bar{R}} \ln H_1 = 0 \tag{3.14}$$

Once this constraint is solved and F_{MNPy} is known, $F_{M\bar{N}\bar{P}y}$ can be obtained immediately from (3.13) and the four-form will then be completely determined.

3.3 Solutions to $\delta\Psi = 0$

It is not a trivial task to solve the constraint (3.14) and thereby obtain the most general expressions for all field strength components. From now on we will restrict ourselves to a subcase, perhaps the simplest possible one, in which (3.14) is satisfied by setting $F_{MNPy} = 0$. While doing so, we must keep in mind that the expressions we obtain follow only from the requirement of supersymmetry preservation; they cannot be said to comprise a supergravity solution until we impose the equations of motion $dF = 0$ and check that Bianchi identity $d * F + F \wedge F = 0$ is also satisfied.

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Assumptions. The vanishing of F_{MNPy} implies that

$$\boxed{\partial_{\bar{R}} \ln G + 8\partial_{\bar{R}} \ln H_1 = 0}$$

This can be then substituted into the expressions found in the previous section in order to yield the set of equations given below

Metric constraint. The hermitean metric is subject to the constraint

$$\boxed{\partial[H_1^2 H_2\ \omega \wedge \omega \wedge \omega] = 0}$$

Field strength components. The four-form is specified by the following components

$$\boxed{\begin{aligned}
F_{01\bar{R}y} &= \frac{1}{2}\partial_{\bar{R}}({H_1}^2 H_2) \\
F_{01M\bar{N}} &= \frac{H_1}{2H_2}\partial_y(H_1 G_{M\bar{N}}) \\
F_{M\bar{P}\bar{Q}y} &= \frac{1}{2}[\partial_{\bar{Q}}(H_2 G_{M\bar{P}}) - \partial_{\bar{P}}(H_2 G_{M\bar{Q}})] \\
F_{MN\bar{P}\bar{Q}} &= \frac{1}{6H_2 H_1^5}\partial_y[H_1^5(G_{P\bar{N}}G_{M\bar{Q}} - G_{M\bar{N}}G_{P\bar{Q}})]
\end{aligned}}$$

All other components vanish.

Killing spinors. The Killing spinor χ can be obtained from (3.5) using

$$2\partial_{\bar{R}} \ln \alpha = \partial_{\bar{R}} \ln H_1\,.$$

4. A consistency check

If this work is to be considered a step forward, it should contain the information we already knew and also expand upon our previous knowledge. At the very least then, we should be able to recover the supergravity solution constructed via the harmonic function rule. Before we can show that this is so, we must first remind ourselves of what the harmonic function rule said in the first place.

4.1 What we knew

M5 $\perp$ M5 (1). Consider an brane configuration made up of two flat M5-branes which intersect in a string. Let them have worldvolumes $01s\bar{s}u\bar{u}$ and $01v\bar{v}w\bar{w}$ respectively, where s, u, v, w are the holomorphic coordinates spanning $\mathbb{C}^4$. If the harmonic functions corresponding to the two branes are denoted by H_A and H_B, the harmonic function rule dictates that the metric should take the form [7]

$$\begin{aligned}
ds^2 &= {H_A}^{-\frac{1}{3}}{H_B}^{-\frac{1}{3}}(-dX_0^2 + dX_1^2) + {H_A}^{-\frac{1}{3}}{H_B}^{\frac{2}{3}}(dsd\bar{s} + dud\bar{u}) \\
&\quad {H_A}^{\frac{2}{3}}{H_B}^{-\frac{1}{3}}(dvd\bar{v} + dwd\bar{w}) + {H_A}^{\frac{2}{3}}{H_B}^{\frac{2}{3}}dy^2\,.
\end{aligned} \tag{4.1}$$

However, there are a few surprises in store. It turns out that both harmonic functions are independent of the overall transverse direction y, in contradiction to the normal mode of operation of the rule which generally leads the harmonic functions to depend *only* on the overall transverse coordinates! What happens in this case however, is that H_A is a function of v, w, $\bar{v}$, $\bar{w}$ and H_B of s, u, $\bar{s}$, $\bar{u}$. Being harmonic functions, they are solutions to the corresponding flat space Laplace equations:

$$\begin{aligned}(\partial_v\partial_{\bar{v}} + \partial_w\partial_{\bar{w}})H_A &= 0\\ (\partial_s\partial_{\bar{s}} + \partial_u\partial_{\bar{u}})H_B &= 0\end{aligned} \tag{4.2}$$

The non-vanishing components of the field strength are

$$\begin{aligned}F_{s\bar{s}\bar{u}y} &= \frac{1}{4}\partial_{\bar{u}}H_B \qquad & F_{u\bar{u}\bar{s}y} &= \frac{1}{4}\partial_{\bar{s}}H_B\\ F_{v\bar{v}\bar{w}y} &= \frac{1}{4}\partial_{\bar{w}}H_A \qquad & F_{w\bar{w}\bar{v}y} &= \frac{1}{4}\partial_{\bar{v}}H_A\end{aligned} \tag{4.3}$$

Though it remains a mystery why the harmonic function rule metric decides to localize the intersection in the relative transverse directions, it was suggested in [11] that perhaps one of the curious features of this solution could be explained. The proposal was that the lack of dependence on the overall transverse direction signals the presence of a membrane. This membrane was conjectured to stretch between the two fivebranes, intersecting each in a string. The fivebranes are then brought closer and closer together until the membrane collapses and all that is left of it is the string intersection on the fivebranes; this intersection, it was said, should be interpreted as a collapsed membrane.

While this is definitely a possibility, it is probably not the sole possibility. However, we will argue that other options exist later. First let us see how the supergravity solution for the system is modified upon the inclusion of a membrane.

M5 $\perp$ M5 $\perp$ M2 (1). Since the two fivebranes are exactly the same as in the previous configuration and the only addition is that of a membrane with worldvolume $01y$, the corresponding metric is easily written down. If the membrane is characterized by a harmonic function H_C, the metric is [7]

$$\begin{aligned}ds^2 = {} & {H_A}^{-\frac{1}{3}}{H_B}^{-\frac{1}{3}}{H_C}^{-\frac{2}{3}}(-dX_0^2 + dX_1^2) + {H_A}^{-\frac{1}{3}}{H_B}^{\frac{2}{3}}{H_C}^{\frac{1}{3}}(dsd\bar{s} + dud\bar{u})\\ & {H_A}^{\frac{2}{3}}{H_B}^{-\frac{1}{3}}{H_C}^{\frac{1}{3}}(dvd\bar{v} + dwd\bar{w}) + {H_A}^{\frac{2}{3}}{H_B}^{\frac{2}{3}}{H_C}^{-\frac{1}{3}}dy^2\end{aligned} \tag{4.4}$$

Once more, we find $H_A = H_A(v, w, \bar{v}, \bar{w})$ and $H_B = H_B(s, u, \bar{s}, \bar{u})$. H_C on the other hand, while still independent of y, can depend on all the coordinates in the complex space.

In addition to the components (4.3) which are of course still present, the four-form field strength has two new components, given by

$$F_{01yM} = \frac{1}{2}\frac{\partial_I H_C}{H_C^2} \tag{4.5}$$

(where M takes values in $\mathbb{C}^4$) and its complex conjugate. While H_A and H_B are conventional harmonic functions, in that they obey the flat space Laplace equations, H_C is instead a generalised harmonic function since it obeys the following differential equation

$$[H_A(\partial_s\partial_{\bar{s}} + \partial_u\partial_{\bar{u}}) + H_B(\partial_v\partial_{\bar{v}} + \partial_w\partial_{\bar{w}})]H_C = 0\,. \tag{4.6}$$

It can now be seen that (4.1) and (4.3) are contained within (4.4) and (4.5) and may be obtained simply by putting $H_C = 1$ in the latter equations.

4.2 What we know now

In order to build up our faith in the analysis of section 3, we check for its consistency with the results of the harmonic function rule reviewed above. One way of doing this is to study what emerges from the equations of section 3.3 when we plug in the metric (4.4). Comparing this to the standard form

$$ds^2 = H_1^2 dX_\mu dX_\nu + 2G_{M\bar{N}} dZ^M dZ^{\bar{N}} + H_2^2 dy^2$$

of the Fayyazuddin-Smith ansatz, we find

$$\begin{aligned} H_1 &= (H_A H_B {H_C}^2)^{-\frac{1}{6}} & H_2 &= ({H_A}^2 {H_B}^2 {H_C}^{-1})^{\frac{1}{6}} \\ 2G_{s\bar{s}} &= 2G_{u\bar{u}} = {H_A}^{-\frac{1}{3}} {H_B}^{\frac{2}{3}} {H_C}^{\frac{1}{3}} & 2G_{v\bar{v}} &= 2G_{w\bar{w}} = {H_A}^{\frac{2}{3}} {H_B}^{-\frac{1}{3}} {H_C}^{\frac{1}{3}}\,. \end{aligned} \tag{4.7}$$

These relations can then be substituted in the expressions found in section 3.3 to obtain the following:

Metric constraint. The metric constraint now implies that

$$\partial_u(H_C^{-1} G_{s\bar{s}} G_{v\bar{v}} G_{w\bar{w}}) = 0 \Rightarrow \partial_u H_A = 0 \tag{4.8}$$
$$\partial_s(H_C^{-1} G_{u\bar{u}} G_{v\bar{v}} G_{w\bar{w}}) = 0 \Rightarrow \partial_s H_A = 0 \tag{4.9}$$
$$\partial_v(H_C^{-1} G_{s\bar{s}} G_{u\bar{u}} G_{w\bar{w}}) = 0 \Rightarrow \partial_v H_B = 0 \tag{4.10}$$
$$\partial_w(H_C^{-1} G_{s\bar{s}} G_{u\bar{u}} G_{v\bar{v}}) = 0 \Rightarrow \partial_w H_B = 0 \tag{4.11}$$

So H_A can be a function of only $v, w, \bar{v}, \bar{w}$ and H_B can depend only on $s, u, \bar{s}, \bar{u}$

Field strength components: electric. The four-form field strength has components of the type $F_{01\bar{P}y}$, given by

$$F_{01My} = -\frac{1}{2}\frac{\partial_M H_C}{H_C^2} \tag{4.12}$$
$$F_{01\bar{N}y} = -\frac{1}{2}\frac{\partial_{\bar{N}} H_C}{H_C^2}\,. \tag{4.13}$$

The only other components with indices 01 have the structure $F_{01M\bar{N}}$, as follows

$$F_{01s\bar{s}} = F_{01u\bar{u}} = \frac{1}{2(H_A H_B)^{1/2}}\partial_y\left(\frac{{H_B}^{1/2}}{{H_A}^{1/2}}\right) \tag{4.14}$$
$$F_{01v\bar{v}} = F_{01w\bar{w}} = \frac{1}{2(H_A H_B)^{1/2}}\partial_y\left(\frac{{H_A}^{1/2}}{{H_B}^{1/2}}\right)\,. \tag{4.15}$$

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Field strength components: magnetic. The contributions of the type $F_{M\bar{P}\bar{Q}y}$, taking into account the functional dependence of H_A and H_B on the complex coordinates, are

$$F_{s\bar{s}\bar{u}y} = \frac{1}{2}[\partial_{\bar{u}}(H_2 G_{s\bar{s}})] = \frac{1}{4}\partial_{\bar{u}} H_B \tag{4.16}$$

$$F_{u\bar{u}\bar{s}y} = \frac{1}{2}[\partial_{\bar{s}}(H_2 G_{u\bar{u}})] = \frac{1}{4}\partial_{\bar{s}} H_B \tag{4.17}$$

$$F_{v\bar{v}\bar{w}y} = \frac{1}{2}[\partial_{\bar{w}}(H_2 G_{v\bar{v}})] = \frac{1}{4}\partial_{\bar{w}} H_A \tag{4.18}$$

$$F_{w\bar{w}\bar{v}y} = \frac{1}{2}[\partial_{\bar{v}}(H_2 G_{w\bar{w}})] = \frac{1}{4}\partial_{\bar{v}} H_A \tag{4.19}$$

And lastly, there are components of the form $F_{MN\bar{P}\bar{Q}}$. These are given below

$$F_{us\bar{s}\bar{u}} = \frac{1}{24} H_A^{1/2} H_B^{1/2} H_C^2 \left[\partial_y \left(H_B^{1/2} H_A^{-3/2} H_C^{-1}\right)\right] \tag{4.20}$$

$$F_{vw\bar{w}\bar{v}} = \frac{1}{24} H_A^{1/2} H_B^{1/2} H_C^2 \left[\partial_y \left(H_A^{1/2} H_B^{-3/2} H_C^{-1}\right)\right] \tag{4.21}$$

$$\begin{aligned} F_{vs\bar{s}\bar{v}} &= \frac{1}{24} H_A^{1/2} H_B^{1/2} H_C^2 \left[\partial_y \left(H_A^{-1/2} H_B^{-1/2} H_C^{-1}\right)\right] \\ &= F_{ws\bar{s}\bar{w}} = F_{vu\bar{u}\bar{v}} = F_{wu\bar{u}\bar{w}} \,. \end{aligned} \tag{4.22}$$

4.3 And how it fits together

It is obvious from the above analysis that by requiring supersymmetry to be preserved in the presence of the metric (4.4), we end up with more field strength components than were predicted by the harmonic function rule. Now that we have all the expressions infront of us, a possible solution to this conundrum appears.

The harmonic function rule implicitly assumes that the only non-vanishing field strength components are those which arise from gauge potentials coupling *electrically* to each of the branes in the system. Since the metric (4.4) was supposed to describe two fivebranes and a membrane, the only components of the four-form field strength which were expected to be present were $F_{MN\bar{P}\bar{Q}}, F_{M\bar{P}\bar{Q}y}$, $F_{01\bar{P}y}$ and their complex conjugates. In particular, there was no allowance for terms like $F_{01M\bar{N}}$. As we have seen above, such terms *do* in fact arise and will not vanish, unless this condition is explicitly imposed.

From the expressions above, it is clear that $F_{01M\bar{N}} = 0$ can only be enforced by imposing $\partial_y H_A = \partial_y H_B = 0$. Plugging this restriction into the remaining equations, we find that the field strength components reduce to (4.5) and (4.3). The Bianchi Identity for the four-form implies that $\partial_y H_C = 0$ and in addition requires H_C to obey the curved space Laplace equation (4.6). Hence, by requiring $F_{01M\bar{N}}$ to vanish, we are able to reproduce the expressions found earlier using the harmonic function rule.

5. Justifications

Given the discussion of the previous section, the origin of the y-independence seen in the harmonic function rule now becomes clear. By insisting that the four-form field strength can only have purely magnetic components for each constituent M5-brane, we are unnecessarily restricting ourselves to only a subclass of solutions; those which are smeared in

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the overall transverse direction. Our intuition which dictates that M5-branes couple to the supergravity three-form purely magnetically has held true for all the systems we have encountered so far, but apparently it breaks down here.

The reason for this is in fact quite simple. For all the M-brane systems we had studied up to now, $F \wedge F$ was always zero and this is no longer true. The strange behaviour of the harmonic function rule can also be explained when we recall that this rule was meant to be applied [5] only to configurations where there were no Chern-Simmon terms in the equations of motion for the four-form. It is clear that the non-zero contribution of $F \wedge F$ has far reaching consequences and is hence worth discussing in a bit more detail.

5.1 What $F \wedge F \neq 0$ implies

The six dimensional action describing the world volume dynamics of the M5-brane contains a term which goes like $\int dB \wedge A$. As a result, electric components of the spacetime supergravity three-form A source a flux for the two-form B on the fivebrane.

A worldvolume flux does not arise in the holomorphically wrapped M5-branes considered earlier [1] for the simple reason that in all these configurations the three-form A had purely magnetic indices. This was due to the fact that $F \wedge F$ vanished and the equations of motion for the field strength reduced to $d * F = 0$. For the fivebrane system (1.1) under study in this note however, the situation is somewhat different. Since $F \wedge F$ is no longer zero, the equations of motion for F are non-trivially satisfied only when F has electric components as well as magnetic.

So, as a result of these Chern-Simons contributions to the equations of motion of its field strength, the supergravity solution for the $M5 \perp M5(1)$ system contains contains electric four-form components $F_{01M\bar{N}}$ and F_{01My} which, through the worldvolume coupling described above, source a two-form gauge field on the fivebrane.

5.2 A rule and an exception

In our attempt to provide a self-consistent explanation of the situation, we will now approach the $M5 \perp M5(1)$ system from another point of view and present a different argument for the existence of a world-volume two-form.

Consider a p-brane which has a q dimensional intersection with another p-brane. Since the two branes have the same dimension, this is referred to as a 'self-intersection'. From the point of view of the $(p+1)$ dimensional worldvolume theory, the intersection corresponds to a dynamical object only there is a $(q+1)$ form to which it can couple.

All p-branes contain scalar fields ϕ which describe their transverse motion. The 1-form field strength of these scalars F_1 is the Hodge dual (on the worldvolume) of the p-form field strength F_p of a $(p-1)$ form gauge field A_{p-1}, i.e.

$$d\phi = F_1 = *F_p = *dA_{p-1} \,.$$

Since the gauge field A_{p-1} couples to an object with $(p-2)$ spatial directions, a p-brane can have a $(p-2)$ dimensional dynamical self intersection [3].

Hence, supergravity solutions corresponding to multiple othogonally intersecting M5-branes can be constructed simply by ensuring that the branes are aligned in such a way that each pair of M5-branes has a 3 dimensional spatial intersection. So when two M5-branes intersect along a string, it appears that the intersection will be non-dynamical and for this reason, the intersection $M5 \perp M5(1)$ was declared to be forbidden, or problematic, or an overlap rather than an intersection; at all events, the consensus seemed to be that this was a system best left alone.

However, a closer inspection shows that these problems might be the result of our somewhat restrictive assumptions. In the derivation of the $(p-2)$ rule we have assumed implicitly that the only fields present in the world-volume theory are scalar fields. If we relax this assumption, the one-dimensional intersection of two M5-branes can be made dynamical by turning on a two-form B on the worldvolume. In the presence of such a two-form, there seems to be no reason to rule out the possibility of the previously forbidden intersection $M5 \perp M5(1)$.

An alternate 'derivation' of the self-intersection rule is based on the BPS $\rightarrow$ no-force argument [4]. Orienting branes of the same type so that they exert no force on each other, as must be the case for stable configurations, it is found that each pair of branes must share $(p-2)$ spatial directions. Applying this logic to the case it hand, it would appear that two M5-branes, oriented such that M5 $\perp$ M5(1), would exert a non-zero force on each other. The system can only be made stable if a flux is introduced to cancel this force. While an explicit calculation still needs to be carried out, we propose that the flux needed to stablise the system is precisely the flux supplied by the two-form B.

5.3 The membrane connection

The only other M-brane configuration in which the constituent branes have a string intersection is $M2 \perp M5(1)$. Because the two M-branes have different dimensions, this system is exempt from the $(p-2)$ rule and there is no apparent conceptual problem with the existence of such an intersection.

A link between the configurations $M2 \perp M5(1)$ and $M5 \perp M5(1)$ was hinted at in [11] where it was conjectured that a pair of M5-branes overlapping in a string was in fact the limiting case of a membrane-fivebrane system, in the limit where the membrane stretched between two overlapping M5-branes has collapsed. This interpretation provided us with a way to avoid confronting the problematic $M5 \perp M5(1)$ intersection directly. Rather than treating it as an entity in itself, we dealt with it only as one particular limit of a system built up from a pair of the already known and understood $M2 \perp M5(1)$ configurations.

In light of the discussions presented above, we propose that the $M2 \perp M5(1)$ and $M5 \perp M5(1)$ system *are* in fact linked, and that the real link between these configurations lies in the two-form which lives on the worldvolume of the fivebrane in both cases. Since it was already known that a worldvolume two-form must exist on the fivebrane in order for the intersection $M2 \perp M5(1)$ to be dynamical, we suggest the presence of an intermediate membrane in the $M5 \perp M5(1)$ system was only postulated in an effort to explain the presence of three-form flux in a purely fivebrane configuration.

Though this flux was not clearly visible in the harmonic function rule analysis [7] of the $M5 \perp M5(1)$ system, its existence behind the scenes was manifested in certain strange effects — the lack of dependence on the overall transverse direction y, for instance. By

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adding a membrane for which y is a world-volume coordinate, we were able to explain away the y-independence of the supergravity solution. The analysis carried out here, on the other hand, makes the background flux transparent and we are able to obtain explicit expressions for electric components of the four-form field strength. We see that $M5 \perp M5(1)$ is a perfectly sensible system on its own, as long as a two-form lives on its worldvolume, and that it is not necessary to resort to the introduction of a membrane in order to make this configuration consistent. Flux for a two-form gauge field is all that is needed and this is perfectly capable of existing even in the absence of a membrane.

6. The hydra-ness of it

In some ways, this M5-brane system is rather like Hydra, the monster in Greek mythology which grew two heads when one was chopped off; every question we attempt to answer leaves many new questions in its stead.

The most obvious question facing us now is to find more general solutions of the supersymmetry preservation conditions, with additional components of the four-form turned on. In particular, by allowing F_{MNPy} to be non-zero we can turn on F_{suvw} and modify the expression for $F_{MN\bar{P}\bar{Q}}$, leading to a far more complicated structure for the field strength than has been considered in this note.

Another avenue to explore is the dimensional reduction of the M-Theory system to an intersecting brane configuration in type IIA and IIB. We would expect the issues that show up in the M5 $\perp$ M5(1) to have analogues in ten dimensions and indeed this is so. As an example, consider the following. Beginning with (1.1), reducing along X^1 and relabelling the coordinates, we obtain the following IIA configuration.

	0	1	2	3	4	5	6	7	8	9
D4	×	×	×	×	×					
D4	×					×	×	×	×	

(6.1)

T-Dualities along $X^1X^2X^3$ and X^4 take us to

	0	1	2	3	4	5	6	7	8	9
D0	×									
D8	×	×	×	×	×	×	×	×	×	

(6.2)

It was shown in [12] that this brane system is stable and supersymmetric only when a background flux is turned on. Recall that all the arguments presented in this note have converged around a central issue: *non-zero electric components for the four-form field strength are needed in order for M5 $\perp$ M5(1) to be an allowed intersection.* We suggest that the flux needed to stabilise the D0 $\perp$ D8 brane configuration in type IIA is simply the lower dimensional (compactified) manifestation of these four-form components.

Many intersecting brane systems can be obtained in string theory by reducing the M-Theory system (1.1) to ten dimensions and performing a series of dualities. We list some examples below.

type IIA	$D4 \perp D4(0)$	$D6 \perp D2(0)$	$D8 \perp D0(0)$
	$NS5 \perp D4(1)$	$NS5 \perp D6(3)$	$NS5 \perp D8(3)$
type IIB	$D5 \perp D3(0)$	$D7 \perp D1(0)$	
	$NS5 \perp D3(0)$	$NS5 \perp D6(2)$	$NS5 \perp D7(4)$

(6.3)

As they stand, we expect these intersecting brane systems to be unstable and/or non-supersymmetric. It should, however, be possible to make these systems stable by turning on a suitable background flux and a purely string theoretic calculation should enable us to work the flux needed. On the other hand, if we have traced the origins of the stability criterion correctly, it should also be possible to arrive at the same result by starting out from M-Theory. If we follow the four-form field strength for the M-brane system (1.1), tracing it through compactification and the relevant dualities needed to arrive at a particular string theory system, we should reproduce the background flux which string theory says is needed to stablise the ten dimensional brane configuration arising at the end of the duality chain. Performing these calculations and verifying this conjecture is another task we have ahead of us.

Yet another question which naturally springs to mind is the connection of the M-brane system (1.1) to calibrations. An extension of the concept of calibrations [13] has been proposed for branes with non-trivial world-volume fields [14]. It would be interesting to explore this further[2] and attempt to find calibrated forms in the background generated by holomorphic M5-brane wrapped on a four-cycle in $\mathbb{C}^4$.

So, one head of this monster has been cut down only to be replaced by many others. To avoid feeling overwhelmed, remember that we had to begin the battle somewhere! The little skirmish outlined above has helped us gain a feel of the way things work in this M-brane system. Although we are not claiming to have won a decisive victory yet, we are confident that we have learnt a lot about the enemy and that our knowledge will prove useful in the future. For now, we will continue our quest, hoping that a day will dawn when this Hydra finally runs out of heads.

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[2] Just as this paper was being submitted, reference [15] appeared which also discusses M5-branes with a worldvolume two-form, in the context of generalised calibrations.

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